Quiz 2 Solutions for 12 PM - 1 PM

1. Use the minimum and maximum values of $f(x) = \sqrt{9+10x^2}$ on the interval $[-2, 1]$ to find numbers A and B such that $A \leq \int_{-2}^{1} f(x)dx \leq B$.

We first find the critical points of f. We have

$$f'(x) = \frac{20x}{2\sqrt{9+10x^2}} = \frac{10x}{\sqrt{9+10x^2}}$$

and

$$f''(x) = \frac{10\sqrt{9+10*x^2} - 10x\frac{10x}{\sqrt{9+10x^2}}}{9+10x^2}$$

We have $f'(x) = 0$ at $x = 0$ and $f''(x) > 0$ hence this is a minimum. We also have $f(-2) = 7$ and $f(1) = \sqrt{19}$ hence we get

$$9 = 3 \cdot 3 \leq \int_{-2}^{1} f(x)dx \leq 3 * 7 = 21$$

2. Suppose that $F(x) = \int_{2}^{\frac{1}{x}} \sqrt{7+2t+4t^2}dt$. Find $F'(2)$.

We have $F'(x) = \sqrt{7+\frac{2}{x}+\frac{4}{x^2}} - \frac{1}{x^2}$ so $F'(2) = -\frac{3}{4}$.