Quiz 1 Solutions for 12 PM - 1 PM

1. Approximate the area A under the graph of $y = 2x + 6x^2$ and over the interval [2,5] by using a Riemann sum with $N = 3$ subintervals. For the choice of points, use the right endpoint for the interval on the left, the middle for the interval in the middle and the left for the interval on the right.

The length of our interval is 1, so we evaluate at the points 3, $\frac{7}{2}$ and 4 and get

$$6 + 54 + 7 + 6\frac{49}{4} + 8 + 96 = \frac{489}{2}$$

2. Calculate the area A of the preceding problem exactly.

We use the right endpoints to define the Riemann sum of n terms and first determine the area from 0 to the point t. Then our sum is

$$\sum_{i=1}^{n} \frac{t}{n}(2 \cdot \frac{ti}{n} + 6 \cdot \frac{t^2 i^2}{n^2}) =$$

$$\frac{2t^2}{n^2} \sum_{i=1}^{n} i + \frac{6t^3}{n^3} \sum_{i=1}^{n} i^2 =$$

$$\frac{2t^2 n(n+1)}{2n^2} + \frac{6t^3 n(n+1)(2n+1)}{6n^3} =$$

$$\frac{t^2 n(n+1)}{n^2} + \frac{t^3 n(n+1)(2n+1)}{n^3}$$

(Since we are taking a limit, there really is no need to cancel any other terms). Taking that limit gives $g(t) = t^2 + 2t^3$. Hence this is the area under our function from 0 to t. So we take $g(5) - g(2) = 255$